MATH 1650: SECTION 5.4: TRANSFORMATIONS WORKSHEET

NAME: _____

1. Let $f(x) = x^2 - x - 6$.

   Match the functional formula on the left with the expression on the right.

   (a) $f(x+1)$

   (b) $f(x)+1$

   (c) $f(x-2)$

   (d) $f(x)-2$

   (e) $f(3x)$

   (f) $3f(x)$

   (g) $f(-x)$

   (h) $-f(x)$

   (i) $x^2 - x - 5$

   (j) $-x^2 + x + 6$

   (k) $x^2 - x - 8$

   (l) $(x+1)^2 - (x+1) - 6$

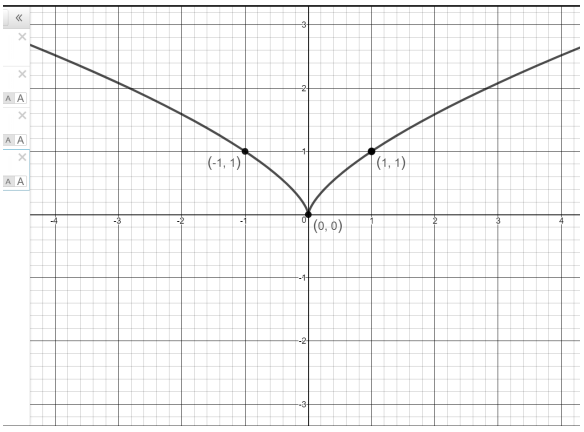
   (m) $9x^2 - 3x - 6$

   (n) $(x-2)^2 - (x-2) - 6$

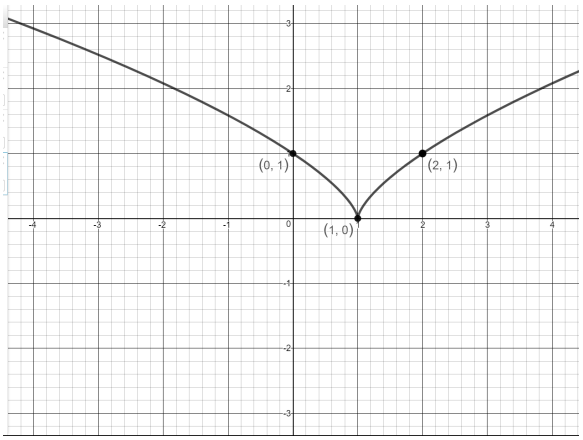
   (o) $x^2 + x - 6$

   (p) $3x^2 - 3x - 18$

2. Consider each pair of graphs below, and answer the following questions:



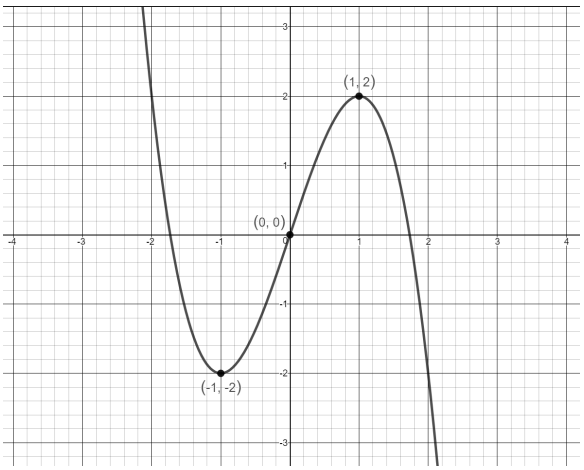
$y = x^{\frac{2}{3}}$



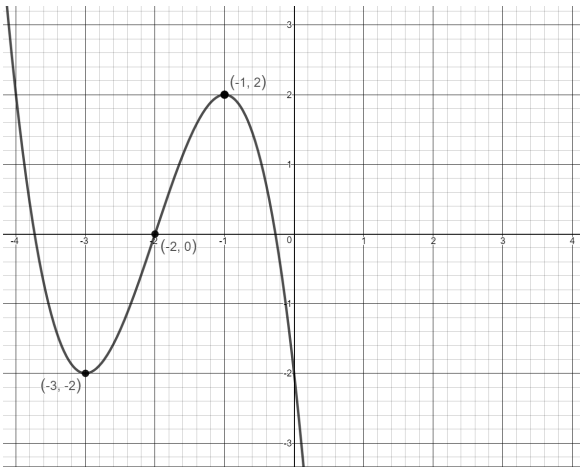
$y = (x-1)^{\frac{2}{3}}$

- How are these two graphs related geometrically?

- How are these two formulas related algebraically?



$y = 3x - x^3$

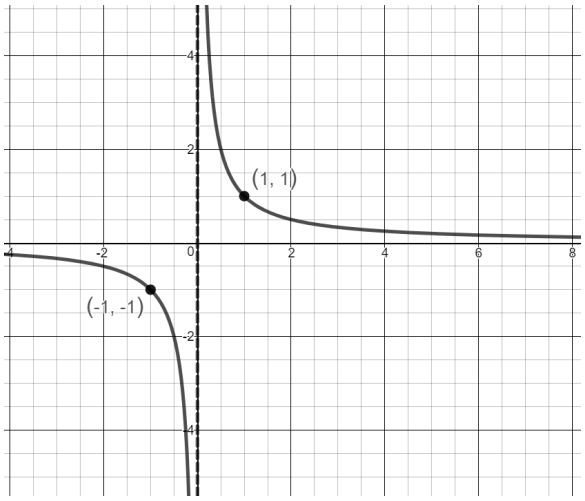


$y = 3(x+2) - (x+2)^3$

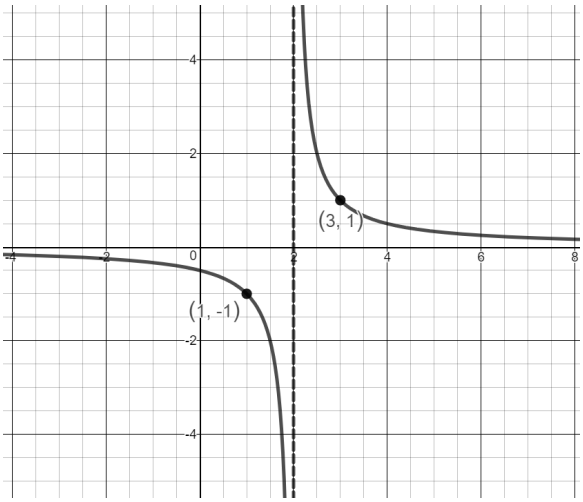
- How are these two graphs related geometrically?

- How are these two formulas related algebraically?

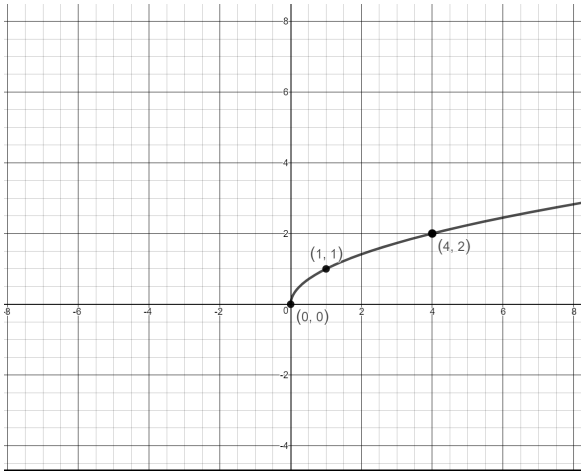
3. Given the formula for the function graphed on the left, find a formula for the function on the right.



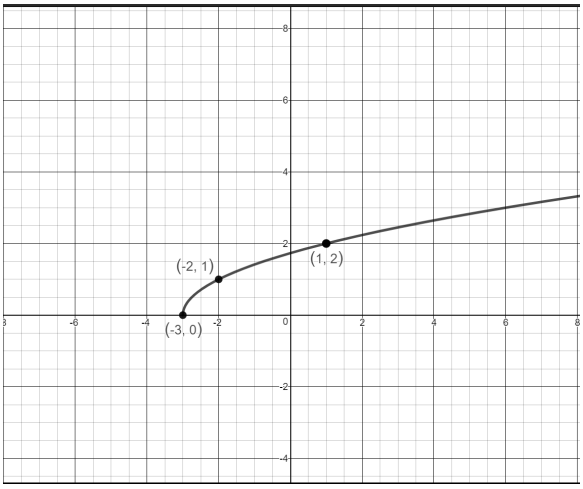
$y = \dfrac{1}{x}$



$y =$



$y = \sqrt{x}$



$y =$

4. If $(3, -4)$ is on the graph of $y = f(x)$, find a point on the graph of:

   (a) $y = f(x - 2)$ (b) $y = f(x + 1)$

5. If (c, d) is on the graph of $y = f(x)$, find a point on the graph of:

   (a) $y = f(x - 2)$ (b) $y = f(x + 1)$

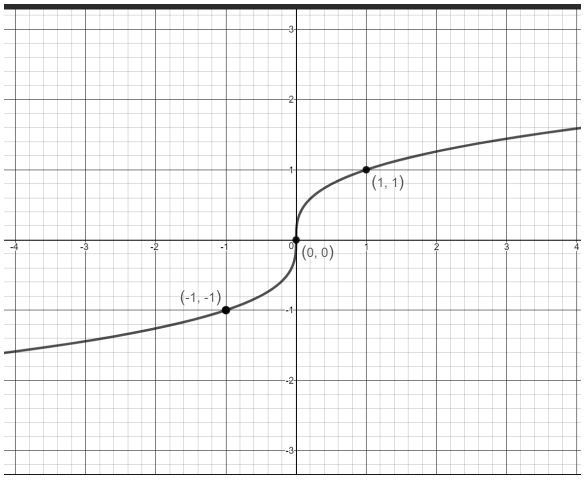
6. If (c, d) is on the graph of $y = f(x)$ and h is a positive number, find a point on the graph of:

   (a) $y = f(x - h)$ (b) $y = f(x + h)$

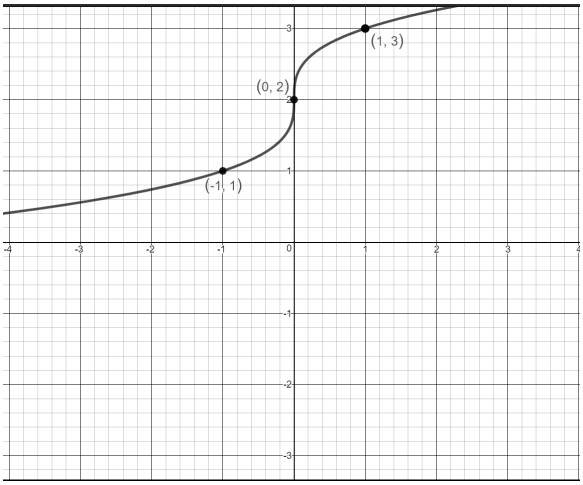
7. If $h > 0$, what is the geometric difference between the graphs of:

   - $y = f(x)$ and $y = f(x - h)$?
   - $y = f(x)$ and $y = f(x + h)$?

8. Consider each pair of graphs below, and answer the following questions:



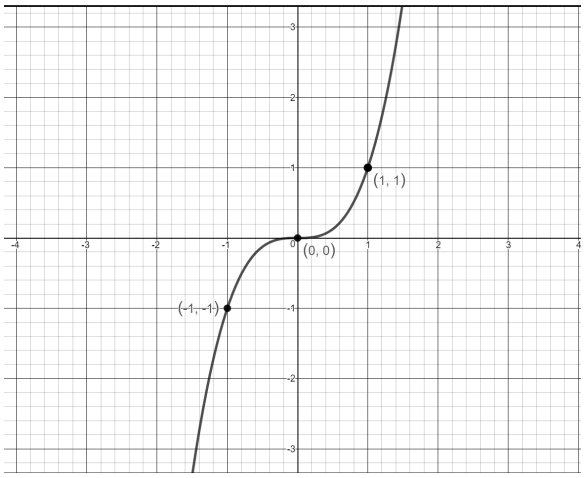
$y = \sqrt[3]{x}$



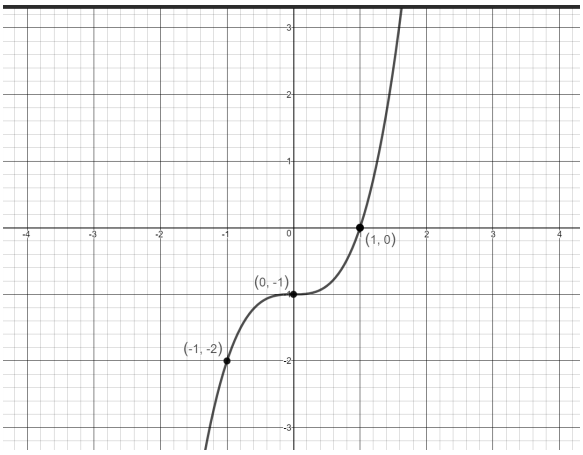
$y = \sqrt[3]{x} + 2$

- How are these two graphs related geometrically?

- How are these two formulas related algebraically?



$y = x^3$

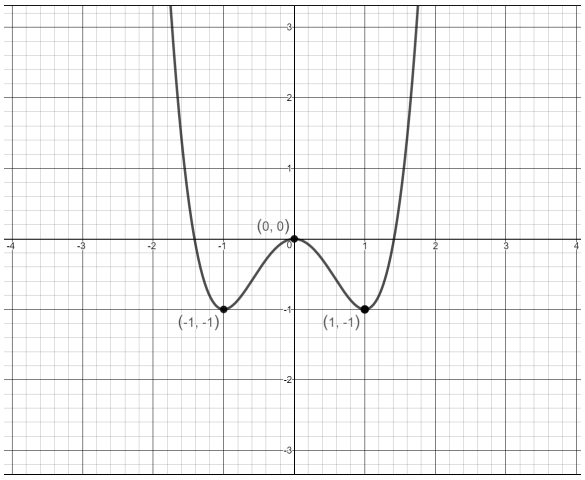


$y = x^3 - 1$

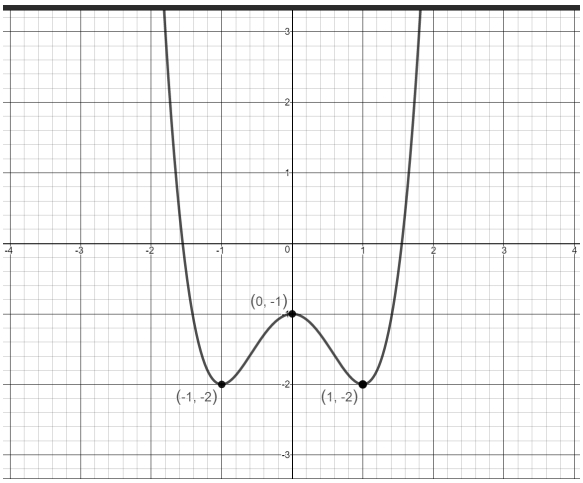
- How are these two graphs related geometrically?

- How are these two formulas related algebraically?

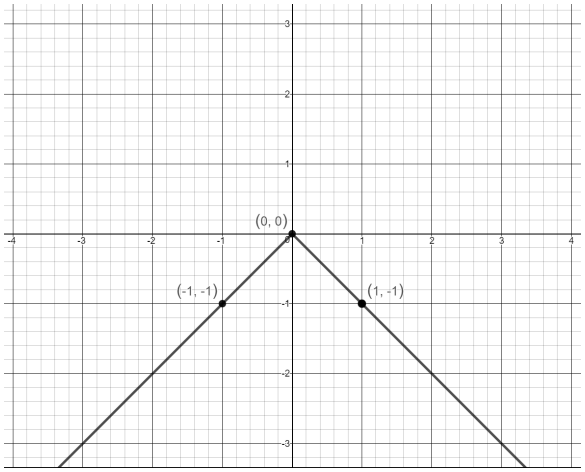
9. Given the formula for the function graphed on the left, find a formula for the function on the right.



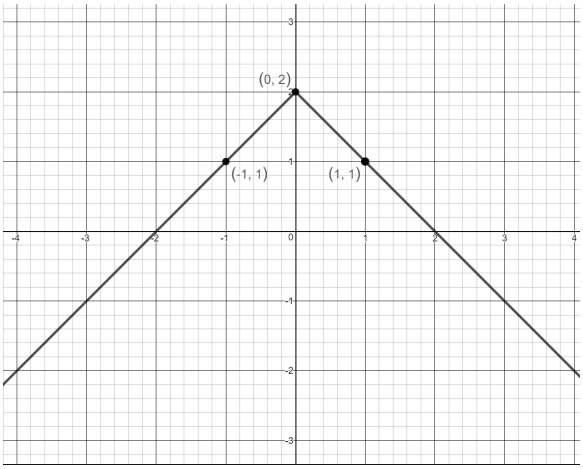
$y = x^4 - 2x^2$



$y =$



$y = -|x|$



$y =$

10. If $(3, -4)$ is on the graph of $y = f(x)$, find a point on the graph of:

    (a) $y = f(x) - 2$ (b) $y = f(x) + 1$

11. If (c, d) is on the graph of $y = f(x)$, find a point on the graph of:

    (a) $y = f(x) - 2$ (b) $y = f(x) + 1$

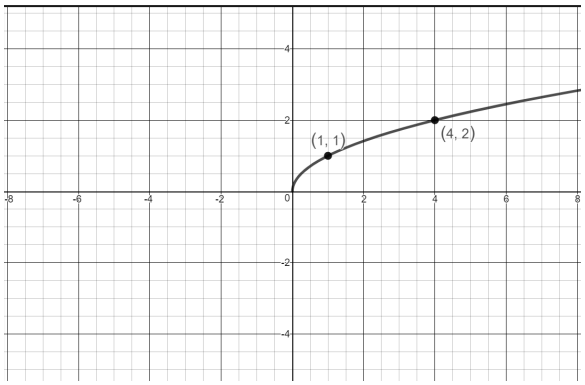
12. If (c, d) is on the graph of $y = f(x)$ and k is a positive number, find a point on the graph of:

    (a) $y = f(x) - k$ (b) $y = f(x) + k$

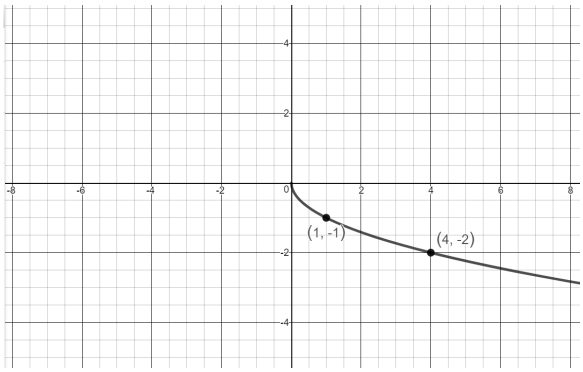
13. If $k > 0$, what is the geometric difference between the graphs of:

    - $y = f(x)$ and $y = f(x) - k$?
    - $y = f(x)$ and $y = f(x) + k$?

14. Consider each pair of graphs below, and answer the following questions:



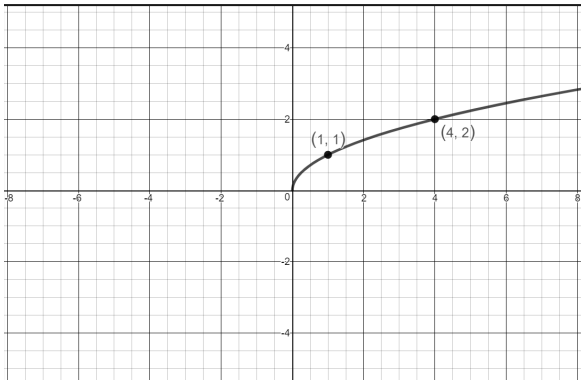
$y = \sqrt{x}$



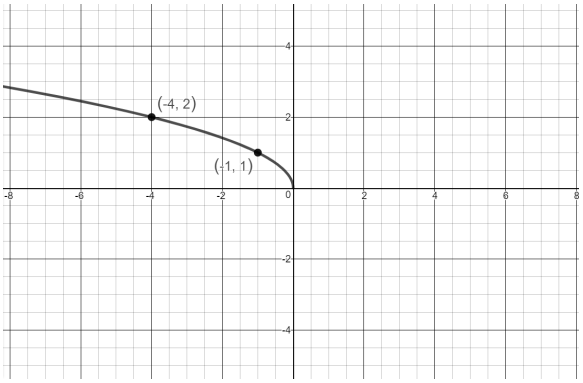
$y = -\sqrt{x}$

- How are these two graphs related geometrically?

- How are these two formulas related algebraically?



$y = \sqrt{x}$

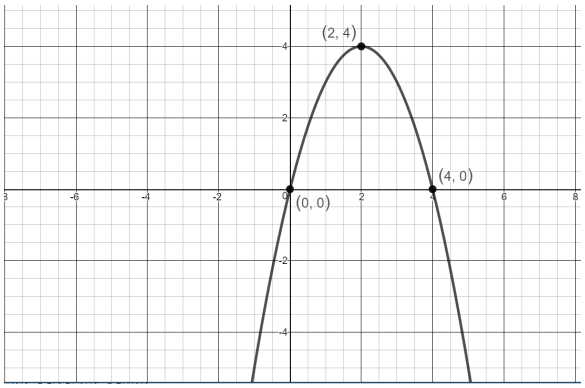


$y = \sqrt{-x}$

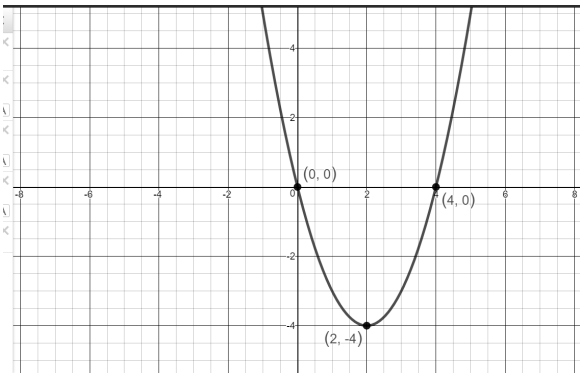
- How are these two graphs related geometrically?

- How are these two formulas related algebraically?

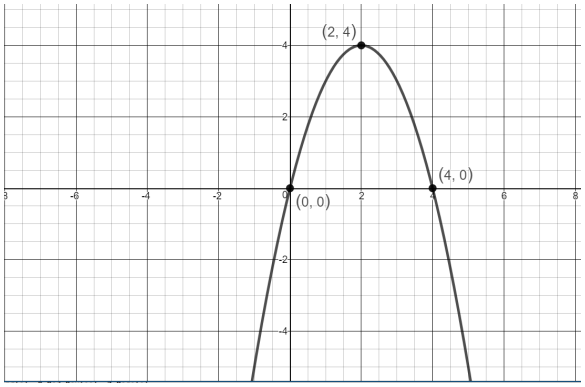
15. Given the formula for the function graphed on the left, find a formula for the function on the right.



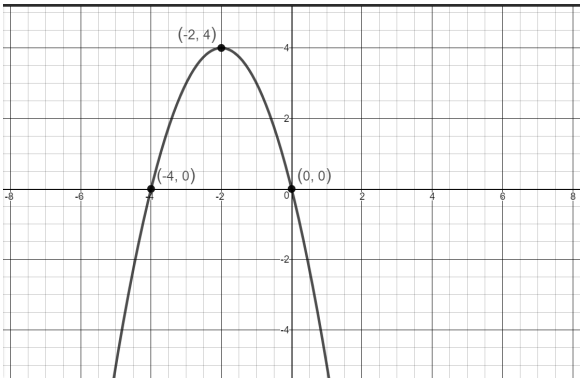
$y = 4x - x^2$



$y =$



$y = 4x - x^2$



$y =$

16. If $(3, -4)$ is on the graph of $y = f(x)$, find a point on the graph of:

    (a) $y = -f(x)$ (b) $y = f(-x)$

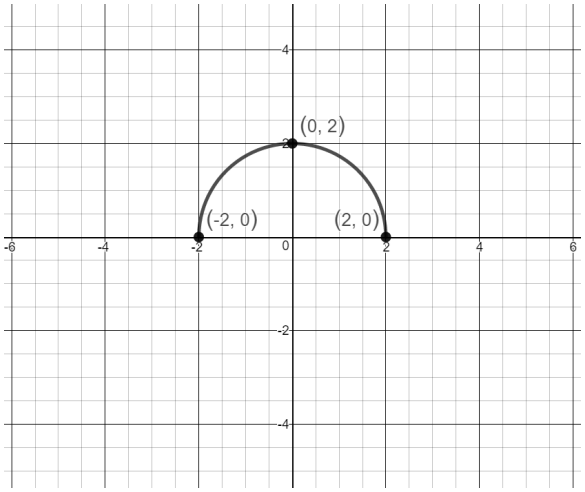
17. If (c, d) is on the graph of $y = f(x)$, find a point on the graph of:

    (a) $y = -f(x)$ (b) $y = f(-x)$

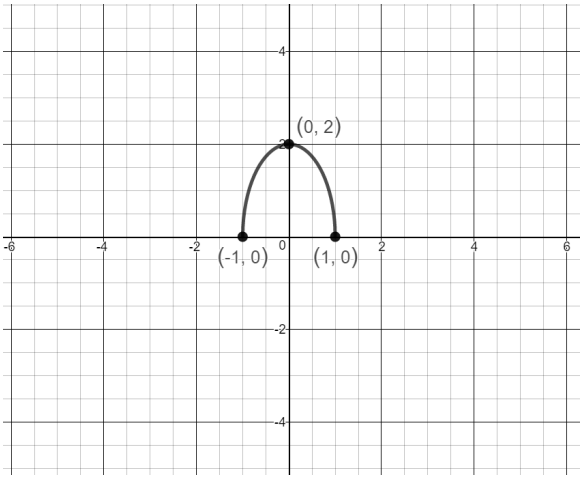
18. What is the geometric difference between the graphs of:

    - $y = f(x)$ and $y = -f(x)$?
    - $y = f(x)$ and $y = f(-x)$?

19. Consider each pair of graphs below, and answer the following questions:



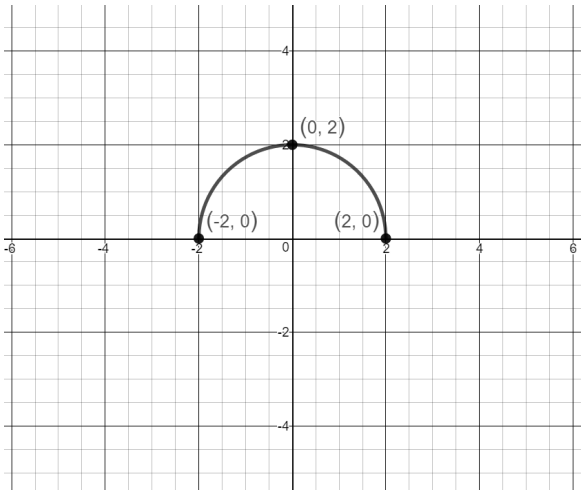
$y = \sqrt{4 - x^2}$



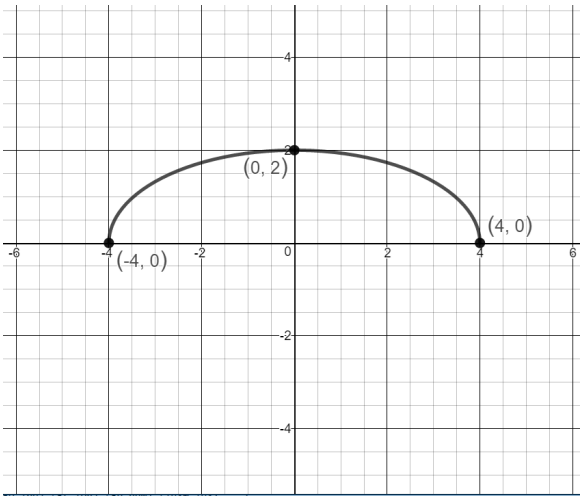
$y = \sqrt{4 - (2x)^2}$

- How are these two graphs related geometrically?

- How are these two formulas related algebraically?



$y = \sqrt{4 - x^2}$

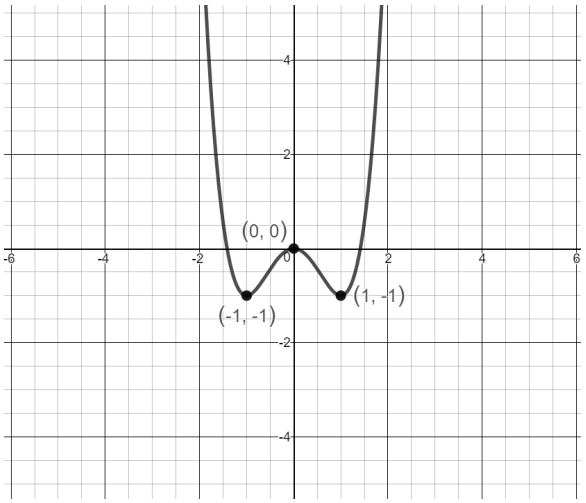


$y = \sqrt{4 - \left(\frac{x}{2}\right)^2}$

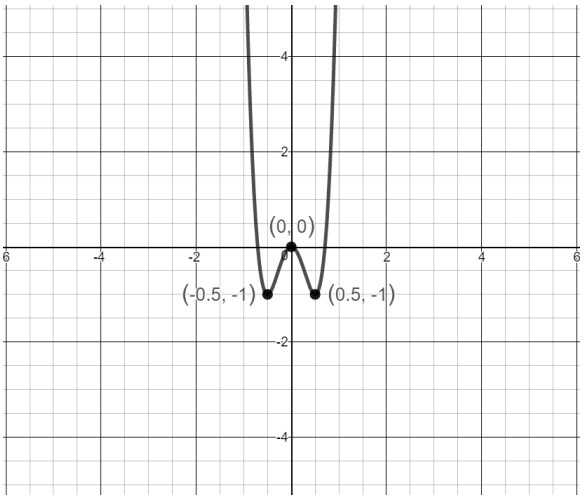
- How are these two graphs related geometrically?

- How are these two formulas related algebraically?

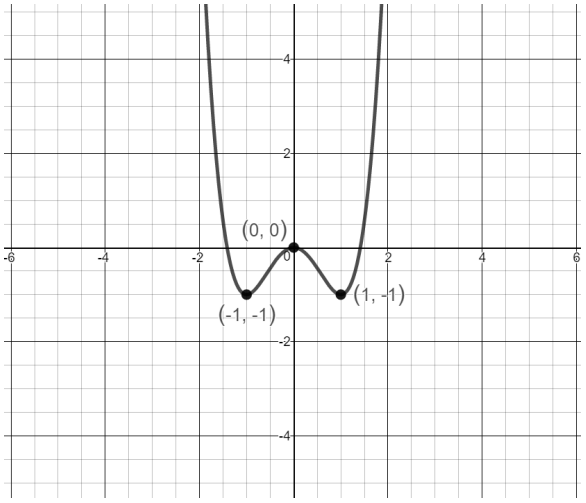
20. Given the formula for the function graphed on the left, find a formula for the function on the right.



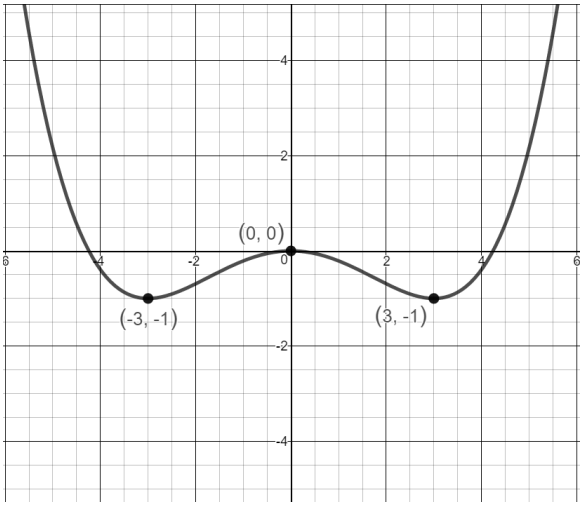
$y = x^4 - 2x^2$



$y =$



$y = x^4 - 2x^2$



$y =$

21. If $(3, -4)$ is on the graph of $y = f(x)$, find a point on the graph of:

    (a) $y = f(5x)$ (b) $y = f\left(\frac{x}{5}\right)$

22. If (c, d) is on the graph of $y = f(x)$, find a point on the graph of:

    (a) $y = f(5x)$ (b) $y = f\left(\frac{x}{5}\right)$

23. If (c, d) is on the graph of $y = f(x)$ and b is a positive number, find a point on the graph of:

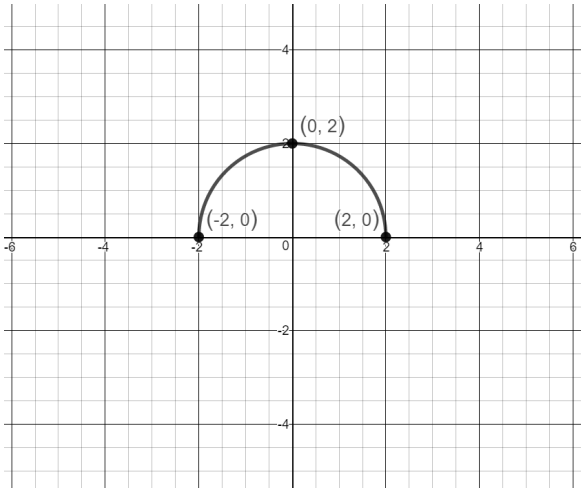
    (a) $y = f(bx)$ (b) $y = f\left(\frac{x}{b}\right)$

24. If $b > 1$, what is the geometric difference between the graphs of:

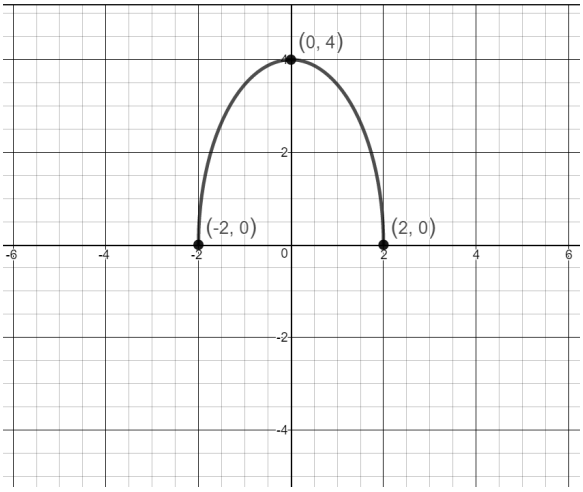
    - $y = f(x)$ and $y = f(bx)$?
    - $y = f(x)$ and $y = f\left(\frac{x}{b}\right)$?

    What happens if $0 < b < 1$?

25. Consider each pair of graphs below, and answer the following questions:



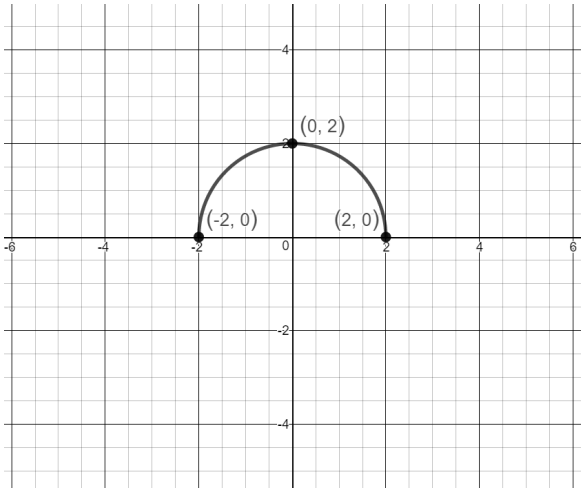
$y = \sqrt{4 - x^2}$



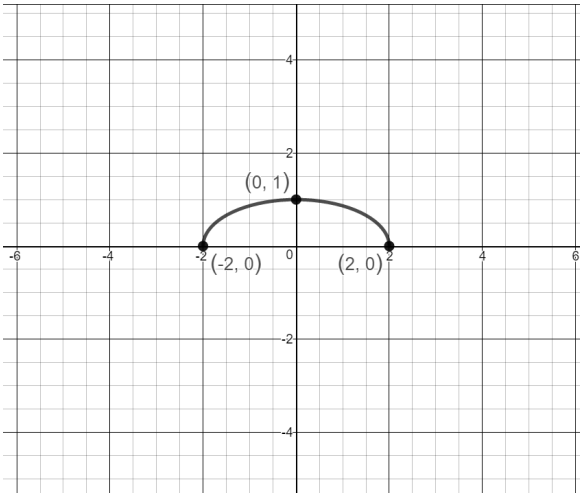
$y = 2\sqrt{4 - x^2}$

- How are these two graphs related geometrically?

- How are these two formulas related algebraically?



$y = \sqrt{4 - x^2}$

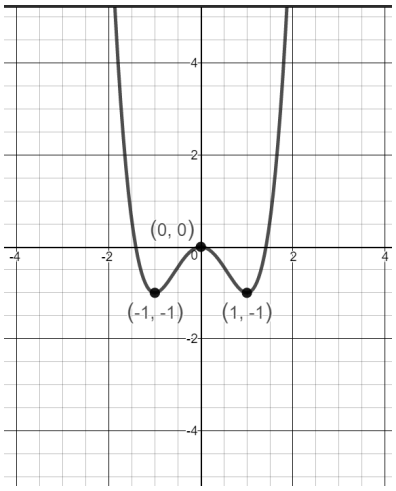


$y = \frac{1}{2}\sqrt{4 - x^2}$

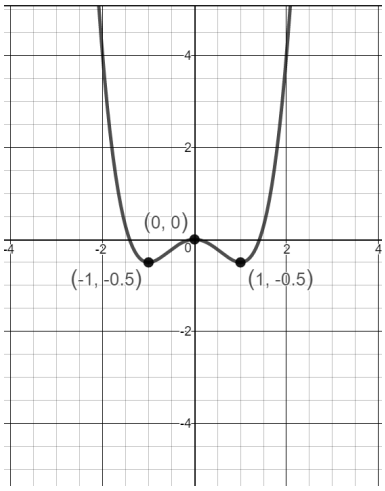
- How are these two graphs related geometrically?

- How are these two formulas related algebraically?

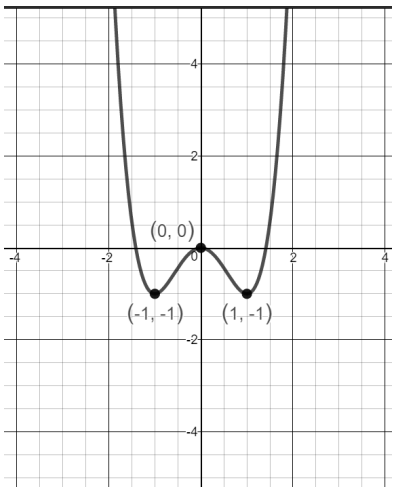
26. Given the formula for the function graphed on the left, find a formula for the function on the right.



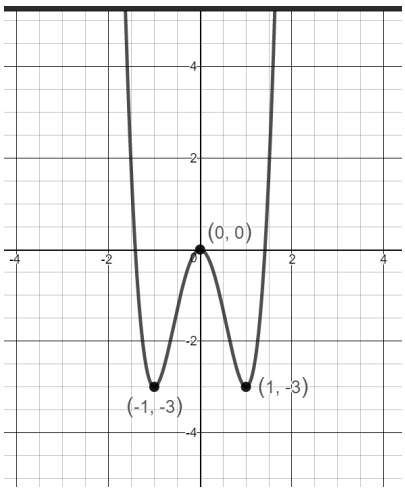
$y = x^4 - 2x^2$



$y =$



$y = x^4 - 2x^2$



$y =$

27. If $(3, -4)$ is on the graph of $y = f(x)$, find a point on the graph of:

    (a) $y = 5f(x)$

    (b) $y = \dfrac{1}{5}f(x)$

28. If (c, d) is on the graph of $y = f(x)$, find a point on the graph of:

    (a) $y = 5f(x)$

    (b) $y = \frac{1}{5}f(x)$

29. If (c, d) is on the graph of $y = f(x)$ and a is a positive number, find a point on the graph of:

    (a) $y = af(x)$

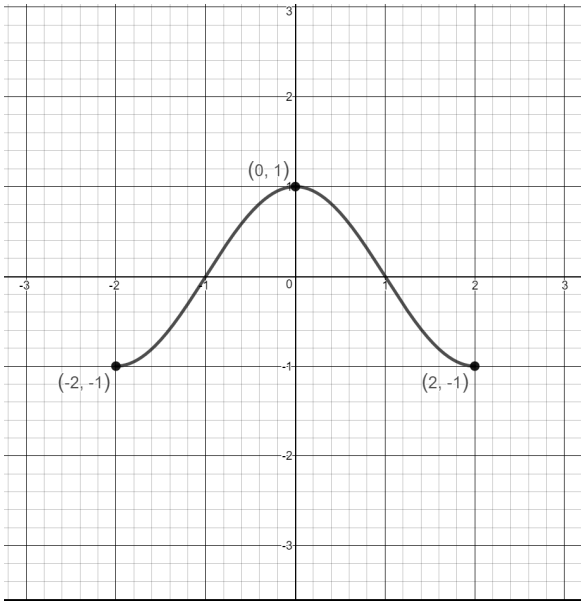
    (b) $y = \frac{1}{a}f(x)$

30. If $a > 1$, what is the geometric difference between the graphs of:

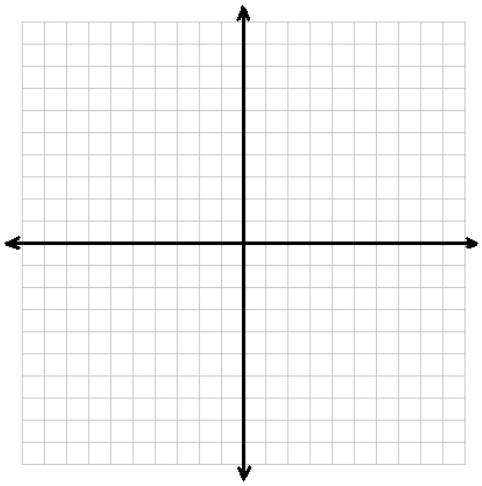
    - $y = f(x)$ and $y = af(x)$?
    - $y = f(x)$ and $y = \frac{1}{a}f(x)$?

    What happens if $0 < a < 1$?

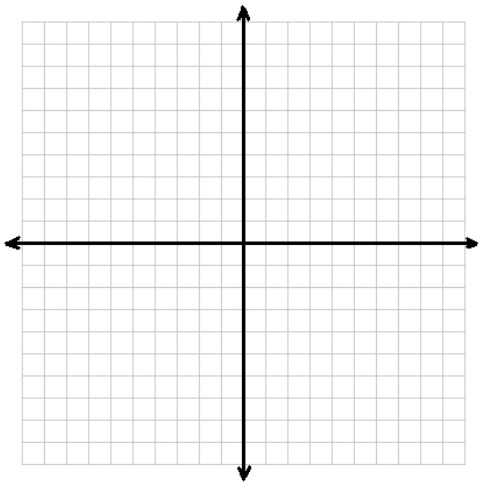
31. Below is the graph of a function $y = C(x)$. Use it to graph each of the functions below. How can you check your answers?



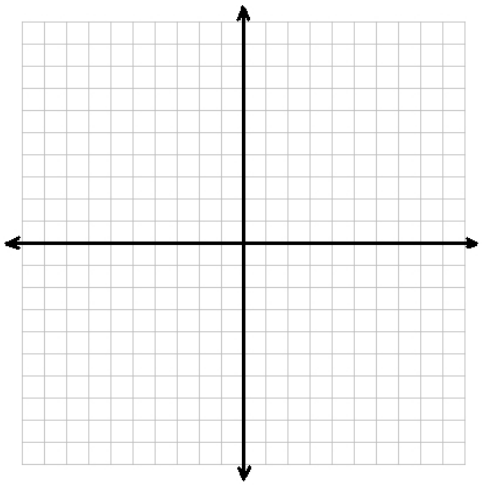
$y = C(x)$



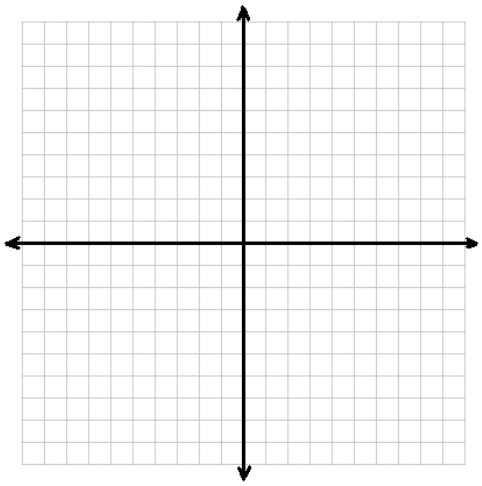
$y = C(x + 2)$



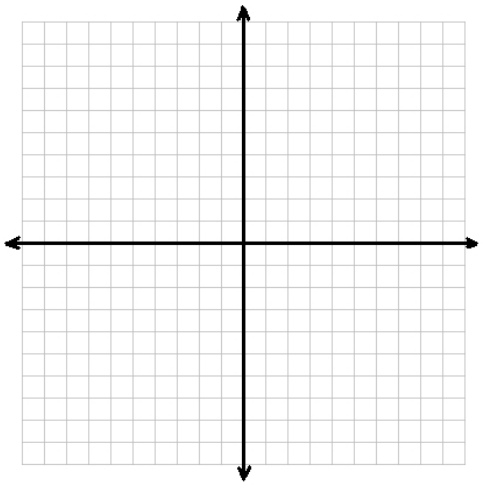
$y = C(x) + 2$



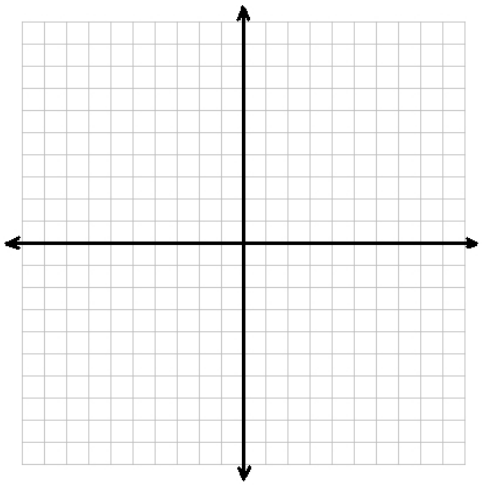
$y = C(2x)$



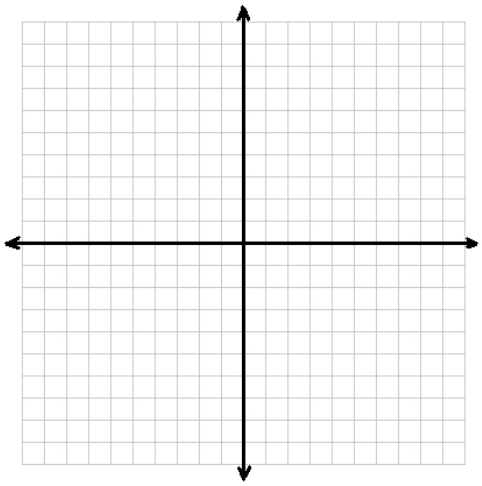
$y = 2C(x)$



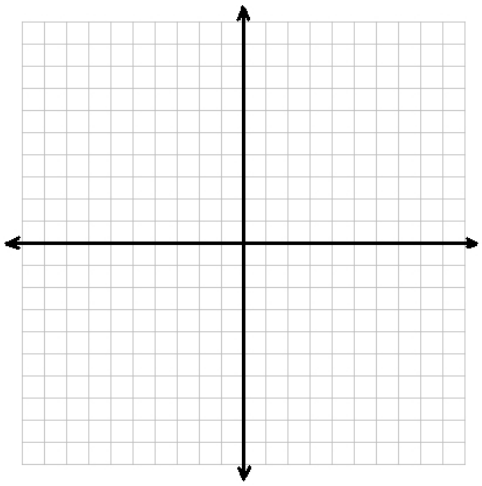
$y = C(-x)$



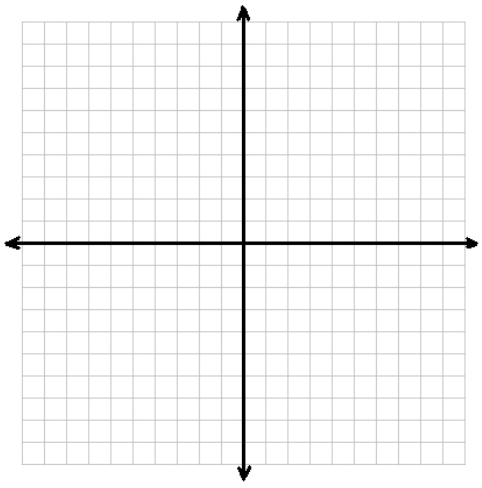
$y = -C(x)$



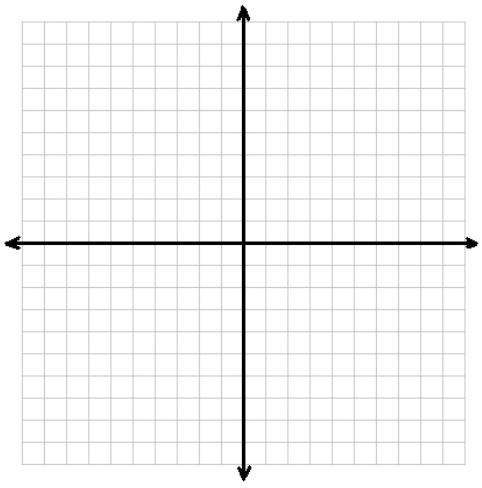
$y = C(x-2)+1$



$y = -C(x+1)$



$y = C(2x+1)$



$y = 2C(x)+1$

HOMEWORK: Section 5.4: 1 - 69 every other odd.